SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Sylow's theorem.

17. Prove that Every finite abelian group is the direct product of cyclic groups.

18. State and prove Eisenstein criterion.

19. Prove that R be a Euclidean ring; then any finitely generated R – module M, is the direct sum of a finite number of cyclic sybmodules.

20. Prove that N is a normal linear transformation on V, then the exists an orthogonal basis consisting of characteristics vectors of N, in which the matrix of N is diagonal. Equivalently, if N is a normal matrix there exists a unitary matrix U such that $UNU^{-1}(=UNU^{X})$ is diagonal.

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APRIL/MAY 2024

GMA11/DMA11 — ALGEBRA — I

Time : Three hours Maximum : 75 marks

Shanmuga Industries Arts & Science College T.V.Malai

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State Sylow's theorem for abelion group.

2. Define class equation.

3. Define Normal subgroup.

4. Define abelian group.

5. Define Polynomials ring.

6. State Einstein criterion.

7. Define Nilpotent transformations.

8. Define Jordan form of a linear transformation.

9. Define Hermitian transformations.

10. Write down the quadratic form for $\begin{bmatrix} 1 & -2 & 4 \\ -2 & 2 & 0 \\ 4 & 0 & -7 \end{bmatrix}$.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) State and prove second Sylow's theorem.

Or

(b) Prove that S_pK has a p – Sylow subgroup.

12. (a) Prove that G is the internal direct product of $N_1,....,N_n$. Then for $i \neq j$ $N_i n N_j = (e)$ and if $a \in N_i, b \in N_j$ then $ab = ba$.

Or

(b) Prove that G be a group and suppose that G is the internal direct product of $N_1,...N_n$. Let $T = N_1 \times N_2 \times N_n$. Then G and T are isomorphic.

13. (a) Prove that ideal $A = (P(x))$ in $[F(x)]$ is a maximal ideal if and only if $P(x)$ is irreducible over F.

Or

(b) State and prove Gauss' Lemma.

14. (a) Prove that for each $i = 1,2,...k_1$ $V_i \neq 0$ and $V = V_1 \oplus V_2 \oplus .... \oplus V_k$. The minimal polynomial of T_1 is $q_i(x)^{t_i}$.

Or

(b) Prove that T in $A_F(V)$ has a minimal polynomial over F the polynomial $P(x) = \gamma_0 + \gamma_1 x + ... \gamma_{r-1} x^{r-1} + x^r$. Suppose further, that V as a module, is a cyclic module. Then there is basis of V over F such that in this basis, the matrix of T is

$$\begin{pmatrix} 0 & 1 & 0 & .... & 0 \\ 0 & 0 & 1 & .... & 0 \\ \vdots & & & & \\ 0 & 0 & 0 & .... & 1 \\ -\gamma_0 & -\gamma_1 & .... & .... & -\gamma_{r-1} \end{pmatrix}.$$

15. (a) Prove that $T \in A(A)$ then given any $V \in V$ there exists an element $W \in V$, depending on V and T such that (uT) equal to (u,w) for all $u \in V$. This element W is uniquely determined by V and T.

Or

(b) Prove that $T \in A(V)$ is Hermitian, then all its characteristic roots are real.